PERFORMANCE MODELING OF SCINTILLATOR-BASED X-RAY IMAGING SYSTEMS

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INTRODUCTION

We recently have been involved in developing x-ray detector systems for industrial NDE applications. A two-step process is commonly used to convert the x-ray flux transmitted through the part under test into a digitized image, with individual pixel values corresponding to the x-ray properties in different regions of the part. First, a scintillator material absorbs x rays and emits light in proportion to the amount of x-ray energy absorbed. Second, this light is converted into an electronic signal by a photodetector device. This electronic signal can then be digitized and stored in a computer system for image display and analysis.

One part of our work involves evaluating the overall performance impact of using various scintillator and photodetector combinations to detect x rays. In our laboratory, where we often evaluate different scintillator devices, we use a CCD camera and a conventional lens to image the light emitted by the scintillators. This simple arrangement provides experimental convenience, since no bonding is required between the scintillator and photodetector, and allows direct comparisons between different scintillator devices. Because of their good electronic noise performance, we use CCD imaging systems from Photometrics Ltd. of Tuscon, AZ. Unlike conventional frame-transfer CCD devices, these systems use a shutter to control an integration period during which light is detected by the CCD, and the resulting image is read off the CCD after the exposure while the shutter is closed.

The obvious drawback in using lenses to couple the scintillation light to the photodetector is the inefficiency of the optical system. Even with relatively wide aperture optics, the optical efficiency is typically on the order of 1 in 1000 for the magnifications of interest to us, meaning that the vast majority of scintillation light photons are wasted. However, we have been able to obtain surprisingly high quality images in reasonably short exposure times, despite the inherently poor optical coupling in our system. Thus we have considered designing a lens-coupled detector system that meets the performance requirements for industrial inspection applications.

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The goal of the work reported here was to better understand the design requirements and the performance limitations of lens-coupled scintillator/photodetector x-ray imaging systems, and to provide a framework for comparison to direct-bonded or fiber-optic coupled systems. Our approach was to apply a statistical treatment to each physical step of the x-ray detection process, producing a mathematical model that can be used to quantitatively predict imaging performance for various system configurations.

DESCRIPTION OF THE MODEL

The detection of x-rays in a scintillator-based imaging inspection system is a multi-step process, as shown in Fig. 1. Neglecting the complicating factors of spatial incoherence and Compton scatter, the underlying processes can be considered in the context of a single pixel in the resulting images. The overall process of x-ray detection is shown schematically in Fig. 1.

Physical Description of Detection

An x-ray source provides a flux of x-ray photons toward the part under test. Some of the x rays interact with the part, causing the incident beam to be partially attenuated. The degree of this attenuation characterizes the part under test, and constitutes the quantity of interest to be measured by the x-ray imaging system.

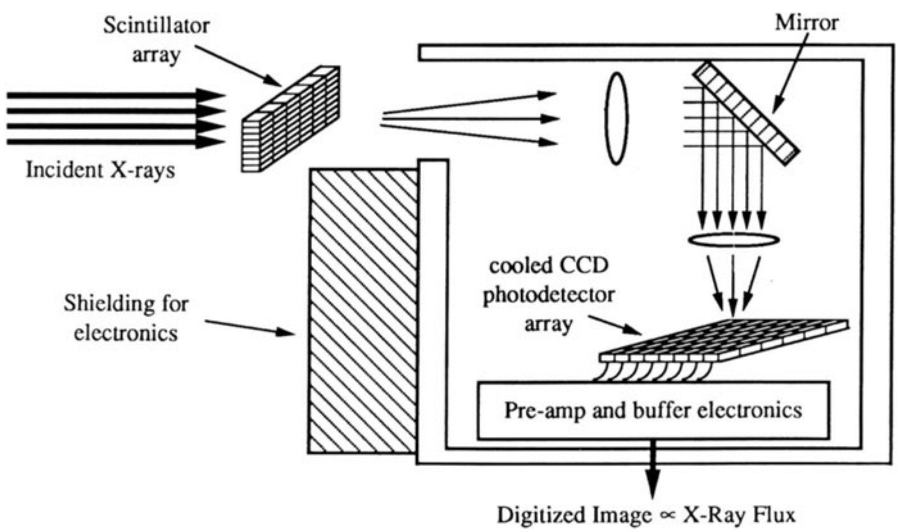

Fig. 1. Example of an lens coupled scintillator/photodetector system for x-ray imaging. X rays deposit their energy in the scintillator device upon detection, resulting in the generation of multiple optical photons. These photons are transferred in a spatially coherent fashion to the photodetector, where they cause electronic excitations. The excitations are measured electronically to form a digital representation of the x-ray properties of samples placed between the x-ray source and scintillator device.

The attenuated x-ray beam then impinges on an element of the scintillator, and some of the x-rays are absorbed through a complicated series of interactions involving the transfer of x-ray energy to the scintillator. The energy absorbed by the scintillator is typically manifested by multiple ionization events, which results in a shower of so-called δ rays [1]. These δ rays are simply excited electrons, some of which relax by generating optical photons characteristic of the scintillator material. The overall result is that a single x-ray with many keV of energy upon detection generates many optical photons, each with energy of a few eV.

Each of these optical photons then either is absorbed within the scintillator device, or escapes from the device. A photon that does escape can be accepted by the optical relay system of lenses and mirrors, or because of the direction in which a particular photon exits the scintillator device, miss the aperture of the optical system and be excluded.

Finally, after the the optical system redirects a photon to the CCD itself, it can be detected or not, depending on whether the optical photon excites an electron across the band gap of the semiconductor material of the CCD. The CCD device can later collect the charges and produce a signal proportional to the number of absorbed photons.

Statistical Description of Detection Processes

The number of x-ray photons N_x emitted by an x-ray source during a period of length t is well described [2,3] by a Poisson distribution of the form

$$P\left(N_x = k\right) = \frac{e^{-\lambda t}\left(\lambda t\right)^k}{k!} \quad , \tag{1}$$

where P is the probability that $N_x = k$ during any particular observation period, and λ is the average rate of x-ray production. In Eq. 1, λt is the characteristic parameter of the distribution.

The Poisson distribution is asymptotically normal (i.e., it approximates a standard Gaussian distribution for large λt) with mean and variance both equal to λt. In other words, if N_x was repeatedly measured over many trials, the average value of N_x would approach λt, and the standard deviation of N_x would approach $\sqrt{\lambda t}$.

In a given observation period, each of these N_x x-ray photons can then either be attenuated in the part under inspection or pass through toward the detector, independent of the others. For example, the probability of any particular x ray being transmitted through the part (under the assumption of a monochromatic x-ray source) can be written as

$$P\left(transmission\right) = p_t = e^{-\alpha L} \quad , \tag{2}$$

where α is the linear attenuation the of the part, and L is the beam path length through the part. Then the probability that k

x rays will be transmitted out of n incident photons is given by a binomial distribution of the form

$$P\left(k \mid n\right) = \binom{n}{k} \left(p_t\right)^k \left(1-p_t\right)^{n-k} \quad . \tag{3}$$

Those photons that do pass through can either be detected (deposit their energy in the scintillator) or not. This process can be similarly modeled with a binomial distribution based on the probability p_s of absorbing a particular photon in the scintillator, which can be expressed as

$$p_s = 1 - e^{-\alpha_s L_s} \quad , \tag{4}$$

where the subscript s indicates scintillator.

The distribution governing the number of optical photons produced in the scintillator by a detected x ray can also be modeled as a Poisson process, since the x ray (and the energetic secondaries it produces) undergo a number of small-cross section interactions in the process of creating a large number of δ rays. The underlying multiple interaction events are analogous in a statistical sense to the multiple interaction that occur when a energetic electron beam bombards an x-ray tube anode to produce x rays. Thus the probability that k δ rays are produced by a detected x ray is

$$P\left(N_\delta = k\right) = \frac{e^{-\mu} \left(\mu\right)^k}{k!} \quad , \tag{5}$$

where μ is the characteristic parameter of the distribution (and the average number of δ rays produced).

All of the other processes leading to detection of an optical photon (conversion of a δ ray into an optical photon, escape vs. internal absorption of the photon in the scintillator, acceptance of an escaped photon into the aperture of the optical system, absorption of the optical photon by the photodetector and the subsequent generation of an electron excitation) are binary decisions, and can be described by binomial distributions with appropriate probabilities.

The Statistical Model

The probability that k x-ray photons are detected by the scintillator can be calculated from the conditional probability of detecting k photons if n were incident, multiplied by the probability that $N_x = n$. Summing each possible term gives

$$P\left(N_d = k\right) = \sum_{n=k}^{\infty} P\left(k \mid n\right) P\left(N_x = n\right) \quad . \tag{6}$$

The sum begins at k, since k photons cannot be detected unless the number emitted from the source was equal to or greater than k.

The distribution describing two independent binomial events is also binomial in form, with a characteristic probability that is the product of the individual process probabilities. The overall conditional probability of detection, $P(k|n)$, is therefore also binomial, with characteristic probability p defined by

$$p = p_t p_s \quad . \tag{7}$$

Substituting the appropriate binomial and Poisson expressions into Eq. 6 yields

$$P(k) = \sum_{n=k}^{\infty} \left[\binom{n}{k} p^k (1-p)^{n-k} \right] \frac{e^{-\lambda t} (\lambda t)^n}{n!} \quad , \tag{8}$$

which after collecting terms, algebraic manipulation, and changing the index of summation reduces to

$$P(N_d = k) = \frac{e^{-p\lambda t} (p\lambda t)^k}{k!} \quad , \tag{9}$$

which is again Poisson in form, but with modified parameter $p\lambda t$. This demonstrates that the conditioning of a Poisson with a binomial distribution is still a Poisson distribution, with characteristic parameter that is the product of the individual distribution parameters.

An analogous argument applies to the second part of the detection process. Clearly, if the description of two binomial processes can be reduced to one, so can several. Therefore, the total probability p_1 of a δ ray causing an excitation in the photodetector is just the product of the probabilities for each step in the process, and a single binomial distribution can model the "detection" of a δ ray. Since the number of δ rays from a detected x ray is described by a Poisson distribution, the number of electrons excited in the photodetector for a single detected x ray, for example, is governed by

$$P(N_{ccd} = k \mid N_d = 1) = \frac{e^{-p_1\mu} (p_1\mu)^k}{k!} \quad . \tag{10}$$

The final step of the model development is to combine the two Poisson distributions that describe the the number of x-rays absorbed in the detector and the number of electrons excited in the CCD for each detected x ray. The resulting distribution is known as compound Poisson [4]. The final result is

$$\begin{aligned} P(N_{ccd} = k,\ k = 1,\ 2,\ \cdots) &= \frac{e^{-p\lambda t} (p_1\mu)^k}{k!} \sum_{j=1}^{\infty} \left(e^{-p_1\mu} p\lambda t\right)^j j^k \\ P(N_{ccd} = 0) &= e^{p\lambda t\left(e^{-p_1\mu} - 1\right)} \end{aligned} \tag{11}$$

Through consideration of the characteristic function of this distribution,

$$\varphi(u) = e^{p\lambda t\left\{e^{p_1\mu\left(e^{iu} - 1\right)} - 1\right\}} \quad , \tag{12}$$

it can be shown that the compound distribution is also asymptotically normal, with mean

$$Mean[N_{ccd}] = p\lambda t \; p_1\mu \tag{13}$$

and variance

$$Var[N_{ccd}] = p\lambda t \; p_1\mu(p_1\mu + 1) \quad . \tag{14}$$

The mean is just the product of the means of the individual Poisson distributions of the first and second parts of the detection process, but the variance has an additional term.

This compound distribution has a well known analog in the so-called insurance problem. The statistics of the number of claims per year and of the dollar amount per claim can not be naively combined to yield a total description of the expected claim amount per year. Instead, a compound distribution is required for an adequate actuarial description.

Conversion from Statistical to Experimental Parameters

The parameters p1 and m used in the statistical model are somewhat cumbersome for experimental comparisons. However, they can be easily substituted for quantities that are easier to measure. For instance, we often choose to recast the product $p_1\mu$ as

$$p_1\mu = n = \frac{PQ}{F} \quad , \tag{15}$$

where P is average number of optical photons that escape the scintillator per detected x ray, Q is the quantum efficiency of the CCD imager, and $1/F$ is the efficiency of the optical imaging system. n is simply a measure of the average number of electrons excited in the CCD per detected x ray.

Splitting p_1 up in this fashion and lumping part of it together with μ is justified by the combinatorial properties of Poisson and binomial distributions discussed earlier, and allows a detailed assessment of the performance impact of each of part of the overall detector system design. For instance, some of the scintillator devices we have evaluated have an angular distribution of output light that is very nearly Lambertian, so the measured optical intensity of scintillation light varies as

$$I(\theta) \propto \cos(\theta) \quad , \tag{16}$$

where θ is the angle measured with respect to the normal of the scintillator output surface. Under this condition the efficiency of a simple single lens optical system (considering only aperture effects and assuming that transmission losses through the system are negligible) is given by

$$\frac{1}{F} = \left[\left(1 + \frac{R}{r}\right)^2 (2\ f\#)^2 + 1\right] \quad , \tag{17}$$

where R and r are the respective object and image pixel sizes (*i.e.*, the size of the scintillator cell that is imaged onto a pixel of the CCD and the dimension of the CCD pixels) and *f#* is just the *f*-stop of the lens.

RESULTS

Eqs. (13-15) can be combined to predict the ultimate signal-to-noise ratio performance of a system as

$$\frac{S}{N} = \frac{Mean\ (N_{ccd})}{\sqrt{Var\ (N_{ccd})}} = \frac{np\lambda t}{\sqrt{n\ (n\ +1)\ p\lambda t}} = \sqrt{\frac{n}{n\ +\ 1}}\ \sqrt{p\lambda t} \quad . \tag{18}$$

Thus the standard result that *S/N* is the square root of the number of the number of x rays detected becomes modified by an optical information loss factor. In practical terms, if n is much less than one, the performance of the system is markedly degraded; but for n reasonably greater than one, the effect is modest.

To compensate for the optical system loss, more x-ray photons must be detected to achieve an equivalent *S/N*. This requires an increase in the integration time to

$$t' = \frac{n\ +\ 1}{n}\ t \tag{19}$$

in order to achieve the same *S/N* as would be obtained without the optical loss. Clearly, if *n* is not greater than one, substantial imaging time penalties are incurred. Since most production inspection systems are ultimately limited in inspection speed and throughput by the amount of x-ray flux available from available sources, in these applications it becomes especially important to ensure that *n* remains greater than one, or that that $P > F/Q$. Thus intrinsically efficient scintillators allow considerably more freedom in the optical system design.

Measurements of *S/N* under various conditions can be used in the context of the model to extract a surprising amount of information about system configuration that is often otherwise difficult to obtain. For instance, simply calculating the standard deviation in a normalized flat field image (*i.e.*, without a part in the x-ray path) and dividing that by the mean field signal yields *S/N*. From Eq. 18,

$$p\lambda t = \left(\frac{S}{N}\right)^2 \frac{n\ +\ 1}{n} \quad , \tag{20}$$

which can be used to verify estimates of tube output flux and scintillator stopping power if *n* is well characterized. It can also be used to estimate integration times required for any required *S/N*. Alternatively, the mean signal *S* is simply

$$S = np\lambda t \frac{1}{G} \quad , \tag{21}$$

where G in the ADC conversion gain (in electrons per ADC count, which is provided by the CCD manufacturer). Combining these expressions gives

$$n = \frac{PQ}{F} = \frac{SG}{\left(\frac{S}{N}\right)^2} - 1 \quad , \tag{22}$$

which provides a direct measure of *n* and hence *P*, which is proportional to scintillator device output.

CONCLUSION

The statistical model we have developed provides a useful framework for evaluating detector performance for x-ray inspection systems, especially in the areas of effective x-ray photon flux utilization, optical output of scintillator devices, and in the inspection speed and throughput limitations imposed by the detector system.

Of course, additional measures of performance (such as resolution, MTF, linearity, radiation damage tolerance, etc.) are also important for describing system capabilities. Further refinement to this model are also indicated, including its extension to include the effects of dark current in the photodetector, electronic read noise, digitization, and of the subsequent data reduction and normalization steps involved in radiographic image production.

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